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Improved Coordination in Nonhierarchic System Optimization

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An improved coordination procedure for the optimization of nonhierarchic systems by decomposition into reduced subspaces is presented. A subspace coordination procedure based on sequential global approximation is extended to include second-order information. This second-order global approximation is formed using information obtained during subspace optimizations. The use of second-order information in the coordination procedure results in improved convergence for nonhierarchic system optimization as compared with the authors' earlier studies using a first-order coordination procedure. The same objective function and cumulative constraints are imposed at each subspace. Nonlocal functions are approximated at the subspaces using global sensitivities. The method optimizes subspace problems concurrently, allowing for parallel processing. Following each sequence of concurrent subspace optimizations, a second-order approximation of the global problem is formed using design data accumulated during the subspace optimizations. The solution of the global approximation problem is used as the starting point for subsequent subspace optimizations in an iterative solution procedure.

I. Introduction

DECOMPOSITION methods are often employed to solve complex engineering design problems involving large numbers of design variables and constraints. Decomposition typically divides engineering design problems into smaller subproblems that are handled by individual design teams using local design expertise. A coordination problem is introduced to direct the overall system design while maintaining the inherent subproblem couplings. In this research an improved procedure for coordination of decomposed nonhierarchic systems is introduced. The nonhierarchic solution procedure proposed in this study can be applied to problems where the responsibility for design optimization is shared by several design teams, each responsible for component design tasks, but restricted by complex design couplings. Typically each team employs a "local" set of design tools (i.e., computational packages and human experience). All teams seek the common goal of optimization and are restricted by the same set of design constraints.

The optimization of nonhierarchic systems using decomposition is a relatively new area of engineering research. Research in hierarchic system decomposition, often referred to as multilevel decomposition, is comparatively well developed.[1] Hierarchic decomposition is characterized by the familiar hierarchic design tree structure. Hierarchic decomposition schemes become difficult to apply when lateral couplings exist between subproblems of the hierarchy. Lateral couplings between branches disturb the hierarchic solution process. Recent studies[2–9] have focused on reformulating these problems in a nonhierarchic format. The decomposed system appears as a network of design nodes (Fig. 1) as opposed to a hierarchic design tree.

The origin of the nonhierarchic decomposition can be traced to the development of the global sensitivity equations (GSE) introduced by Sobieszczanski-Sobieski.[10] The network structure of nonhierarchic system decomposition was proposed by Sobieszczanski-Sobieski with the GSE providing first-order information descriptive of the complex coupling.

Global sensitivities were used effectively in a structures-control optimization study by Sobieszczanski-Sobieski et al.[7] A piecewise linear procedure, based on the GSE, was employed to optimize the nonhierarchic decomposition of structure and controls. The piecewise linear optimization is applied to the full system. In this system-level implementation, local design teams are limited to performing design analysis based on fixed global inputs. Piecewise linear optimization of the full system has been the dominant nonhierarchic algorithm employed to date. Examples include Refs. 5 and 11–13.

Nonhierarchic optimization strategies have been proposed that provide for local design team optimization. These strategies more closely model existing design practices, where local design decisions are accommodated. A strategy of concurrent subspace optimizations was proposed by Sobieszczanski-Sobieski.[9] The subspace optimization strategy provides for local design decisions. A global coordination procedure is required to coordinate subspace design decisions. The Sobieszczanski-Sobieski proposal uses a constraint responsibility and tradeoff scheme for subspace coordination. The Sobieszczanski-Sobieski nonhierarchic algorithm is implemented in Ref. 4.

Another strategy for subspace optimization of nonhierarchic systems is developed by Pan and Diaz.[6] The proposed algorithm is not as aggressive as the Sobieszczanski-Sobieski proposal in that it searches the subspaces sequentially and not concurrently. Although improved solutions result using the Sobieszczanski-Sobieski[9] nonhierarchic algorithm, convergence to an optimum is

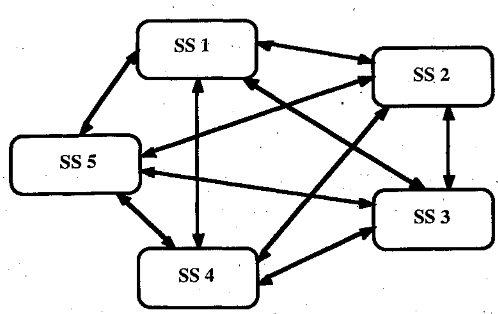

Fig. 1 Nonhierarchic decomposition network structure.

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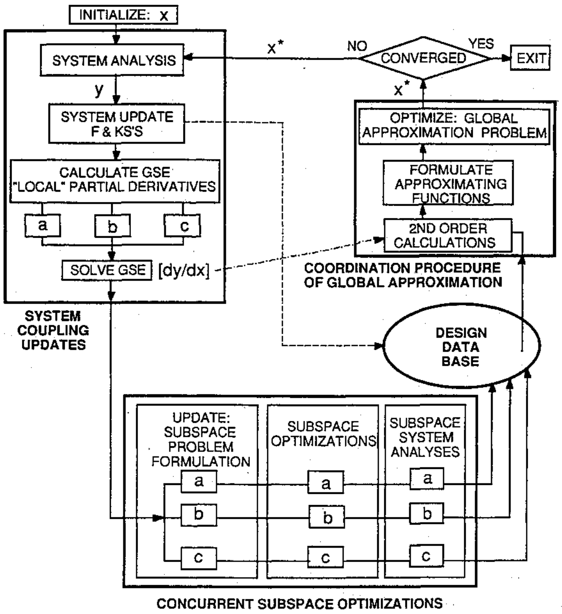

Fig. 2 Algorithm flowchart.

not assured. In the proposal of Pan and Diaz,[6] a proof of convergence is offered (for convex systems). Convergence is achieved as a result of the sequential nature of subspace optimizations combined with a strategy for moving away from "pseudo-optimal" points. The drawback of the proposal of Pan and Diaz[6] is its sequential nature. They point out that the concurrent solution of the subproblems is the ultimate goal in nonhierarchic optimization.

In Renaud and Gabriele[2] a nonhierarchic optimization strategy is proposed that provides for concurrent subspace optimizations. The approach incorporates a global approximation strategy for the coordination of subspace optimization. The global approximations are built using design information generated during subspace optimizations.

The global approximations used in the coordination procedure of Renaud and Gabriele[2] were formed using the accumulated approximation strategy of Rasmussen.[14] The accumulated approximations (Lagrangian interpolation) were built using first-order information for the actual function(s) to be approximated. In this paper a simple nonhierarchic decomposition of Rosenbrock's function[15] illustrates the need for second-order-based approximation in the coordination procedure of global approximation.

The Rasmussen[14] accumulated approximation is modified to include a second-order basis of approximation. Improved performance is observed using the second-order-based coordination procedure of global approximation as compared with the first-order procedure used in Ref. 2. Numerical results from optimizing a nonhierarchic decomposition of the welded beam problem[16] and from optimizing the decomposed Rosenbrock's function illustrate the method's potential.

II. Algorithm Overview

This section presents an overview of the nonhierarchic system optimization algorithm proposed in Ref. 2 and modified in this research. The reader is referred to Fig. 2, which depicts the algorithm flowchart. The method optimizes decomposed subspaces concurrently, followed by a coordination procedure of global approximation. In this study, optimization of a second-order global approximation problem is proposed as an improved coordination procedure for directing system convergence and resolving subspace conflicts. The second-order global approximation problem is formed using information obtained during concurrent subspace optimizations. This corresponds to existing design practice where individual design groups optimize their component designs, and tradeoffs or compromises are made based on the "whole" of the designers' previous experience. In the algorithm proposal, information obtained during the subspace optimizations is used to represent designers' knowledge and data for respective design subspaces. The subspace optimizations serve to improve the component designs of each design group, while simultaneously building a design data base used in the coordination procedure of global approximation. The design data base stores actual function and constraint information for design vectors investigated during subspace optimizations. This information is "free" since it is generated by the local optimizer(s) during the subspace optimizations. The cost of storing the design data base represents additional overhead intrinsic to approximation methods. System coupling is maintained and updated using the GSE approach as introduced in Refs. 9 and 10. The algorithm and each module in Fig. 2 are addressed in greater detail after a brief discussion about notation.

The notation of nonhierarchic system decomposition introduced in Ref. 9 is adapted for use in this research. [Bold face characters denote vectors (e.g., $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{p}, \boldsymbol{g}, \boldsymbol{h}$). Bold face characters with subscripts denote subvectors (e.g., $\boldsymbol{x} = [\boldsymbol{x}_a, \boldsymbol{x}_b, \boldsymbol{x}_c]$, $\boldsymbol{y} = [\boldsymbol{y}_a, \boldsymbol{y}_b, \boldsymbol{y}_c]$, etc.). Non-bold face characters denote individual vector components (e.g., $x \in \boldsymbol{x}$, and $y \in \boldsymbol{y}$, etc.).] The standard form of a nondecomposed optimization problem is represented in Eq. (1):

Minimize:

$$F(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{p}) \tag{1}$$

subject to:

$$g_m(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{p}) \geq 0 \qquad m = 1, 2, \ldots, M$$

$$h_k(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{p}) = 0 \qquad k = 1, 2, \ldots, K$$

where $\boldsymbol{x}$ is a design vector, $\boldsymbol{y}$ is a state vector where state variables $y = y(\boldsymbol{x}, \boldsymbol{p})$, and $\boldsymbol{p}$ is a parameter vector (fixed).

The design vector $\boldsymbol{x}$, state vector $\boldsymbol{y}$, and parameters $\boldsymbol{p}$ serve as the variables in the objective function F and in the constraint functions $\boldsymbol{g}$ and $\boldsymbol{h}$. The nonhierarchic system decomposition partitions the $\boldsymbol{x}$ vector into number of subsystems (NSS) subvectors to be used in separate subspace optimizations. This partitioning is unique with each of the $(i = 1, \ldots, \text{NSS})$ subspaces having its own subvector $\boldsymbol{x}_i$. The partitioning is made based on past design experience, following the existing design partitions of a design organization. The local design expertise associated with a given subspace is referred to as a contributing analysis (CA) (see Ref. 9). The state vector $\boldsymbol{y}$ is similarly partitioned. It is the state subvectors $\boldsymbol{y}_i$ that introduce subspace couplings. Decomposition of the problem in Eq. (1) is depicted in Fig. 3 for three subsystems a, b, and c. The complex coupling between CAs via the state vectors is depicted in the nonhierarchic network of Fig. 3.

In general, the constraints can be treated as state variables in the nonhierarchic optimization problem. This type of definition for state variables has been successfully implemented in Refs. 2, 4, and 7. The reasoning behind defining the constraints as state variables is that the GSE can be used to provide constraint sensitivities directly. A small degree of ambiguity is introduced in Eq. (1) when defining the constraints as state variables. In implementation studies the ambiguity is easily resolved. By declaring constraints as

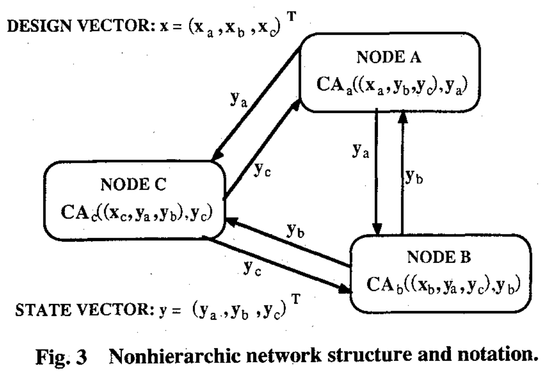

Fig. 3 Nonhierarchic network structure and notation.

state variables, the aforementioned partitioning of the state vector y then includes a partitioning of constraints.

The unique decomposition of the design vector x implies that a design variable $x \in x_i$ cannot appear in another design subvector $x_{j \neq i}$. The state vector partitioning is based on the state analysis capability of each subspace design team. The state analysis capabilities, although unique, are coupled, since a state y of a given state subvector y_i may depend on nonlocal state subvectors $y_{j \neq i}$. By necessity state variables may be used/evaluated in more than one subspace (see Fig. 3). With constraints defined as state variables, the preceding definition allows all constraints to appear in all subproblems. The problem parameters p are not partitioned and can appear in any subspace since they are assumed fixed. The same objective function F is optimized at each subspace with respect to the local design subvector. The concurrent subspace optimizations, design data base, and the modified coordination procedure of global approximation proposed in this research are detailed in the following sections. Although the algorithm includes the system-coupling features of the proposal in Ref. 9, it is distinctively different in terms of the coordination procedure used to drive the subspace optimizations.

III. System Coupling and Subspace Optimization

The algorithm proposed in Ref. 2 retains the feature of "local" subspace optimization, first proposed in Ref. 9. The subspace optimization features used in this research are described later, including the features required for the coordination procedure of global approximation. A nonhierarchic decomposition of the problem depicted in Eq. (1) is assumed to span three subspaces a, b, and c for illustration in this section (where NSS = 3). The design vector and state vector are decomposed into subvectors as shown in Eq. (2):

$$x^T = [x_a, x_b, x_c] \qquad y^T = [y_a, y_b, y_c] \tag{2}$$

The set of analyses performed at each subspace is referred to as a contributing analysis in the overall design problem. Equations (3) along with Fig. 3 illustrate the mathematical nature and interaction of the CAs[9]:

$$\begin{aligned} &\mathrm{CA}_a[(x_a, y_b, y_c), y_a] = 0 \\ &\mathrm{CA}_b[(x_b, y_a, y_c), y_b] = 0 \\ &\mathrm{CA}_c[(x_c, y_a, y_b), y_c] = 0 \end{aligned} \tag{3}$$

The constraints g and h imposed in the original problem of Eq. (1) are incorporated in the state vector y. Therefore they have also been partitioned to the subspaces. The algorithm is initialized at a reference design x^0 with the initial state vector y^0 being evaluated by iteration using Eqs. (3) (system analysis, Fig. 2). Following concurrent "local" partial derivative calculations, the GSE are solved for the system sensitivity derivatives ($\mathrm{d}y/\mathrm{d}x$) (system coupling updates, Fig. 2).

Each subspace ($i = a, b, c$) is then optimized (concurrent subspace optimizations, Fig. 2) with respect to its local design subvector x_i starting at the current design (x^0, y^0). Equations (4–9) detail the subspace optimization problem for subspace a. Each subspace optimization is subject to a local cumulative constraint[17] (e.g., KS_a, with constraints assigned to y_a) and to nonlocal cumulative constraint approximations (e.g., KS_b and KS_c, with constraints assigned to y_b and y_c, respectively). The subspace optimization problems are solved concurrently under move limit (ml) restrictions that maintain the accuracy of the constraint approximations.

The cumulative constraints KS are formed using the Kreisselmeier and Steinhauser[17] function of Eq. (5). The local cumulative constraint (e.g., KS_a) is analyzed directly [i.e., Eq. (5)], where the local inequality constraints ($g_m \in y_a$) are estimated as shown in Eq. (6), where Eq. (6) represents the output for the "local" nonlinear contributing analyses (CA_a) based on approximate inputs of nonlocal states. To account for system coupling, the nonlocal states (e.g., y_b, y_c) used in Eq. (6) are approximated using the GSE generated system sensitivity derivatives ($\mathrm{d}y/\mathrm{d}x$) in a first-order Taylor series approximation. Equation (7) depicts these nonlocal state approximations based on the GSE.

We are reminded that the constraints are defined as state variables y and therefore the respective system sensitivity derivatives provide explicit constraint sensitivity [i.e., $\mathrm{d}g_m/\mathrm{d}x \in (\mathrm{d}y/\mathrm{d}x)$]. Note that in calculating KS_a not all nonlocal state variables need to be approximated using Eq. (7). Only those variables required to calculate y_a in Eq. (6) are approximated. This parallels existing design processes, where individual groups analyze their local constraints based on approximations of neighboring design states. The implementation of Eq. (6) allows individual design teams to more directly estimate "local" state changes due to "local" design changes. This is a departure from strict use of GSE linearized constraints as proposed in Ref. 9. Equation (6) is an attempt to provide more design freedom during subspace optimization. In cases of highly complex local constraint coupling, the user may need to approximate local constraints or local state variables using Eq. (7) in place of Eq. (6) (i.e., the same as Ref. 9):

Minimize:

$$F[x_a, (x_b^0, x_b^0), (y_a, y_b, y_c), p] \quad wrt\ x_a \tag{4}$$

subject to

$$KS_a(y_a, y_b, y_c) \geq 0$$

$$KS_b \geq 0$$

$$KS_c \geq 0$$

$$h_k = 0 \qquad k = 1, 2, \ldots, K$$

$$(1 - \mathrm{ml})\, x_a^0 \leq x_a \leq (1 + \mathrm{ml})\, x_a^0$$

where y_a is from Eq. (6), y_b and y_c are from Eq. (7), KS_b and KS_c are from Eq. (9), h_k is from Eq. (7) if $h_k \in y_{b,c}$ or from Eq. (6) if $h_k \in y_a$, ml is the local move limit ($0 \leq ml \leq 1$), and

$$KS_a = -\left[\frac{1}{\rho} \ln\left(\sum_{m=1}^{M_a} e^{-\rho g_m}\right)\right] \tag{5}$$

where $g_m \geq 0.0$ (standard form) and $g_m \in y_a$

$$y_a \approx f(x_a, x_b^0, x_c^0, y_b, y_c) \tag{6}$$

$$y_i \approx y_i^0 + \left(\frac{\mathrm{d}y_i}{\mathrm{d}x_a}\right)^T (\Delta x_a) \tag{7}$$

where $i = b$ or c, respectively, with $y_b \in y_b$ and $y_b \in y_c$ and

$$\frac{\mathrm{d}KS_i}{\mathrm{d}x} = \left(\sum_{m=1}^{M_i} e^{-\rho g_m}\right)^{-1} \sum_{m=1}^{M_i} \left\{\left[\left(e^{-\rho g_m}\right)\left(\frac{\mathrm{d}g_m}{\mathrm{d}x}\right)\right]\right\} \tag{8}$$

where $i = b$ or c, respectively, and $g_m \in y_b$ or y_c respectively, and

$$KS_i \approx KS_i^0 + \left(\frac{\mathrm{d}KS_i}{\mathrm{d}x_a}\right)^T \Delta x_a \tag{9}$$

The nonlocal cumulative constraints KS_b and KS_c are approximated using a first-order Taylor series approximation of the KS function as shown in Eq. (9). This avoids approximating all of the nonlocal constraints individually. The gradient of the KS function is easily calculated using Eq. (8) for each design variable x. Each term (dg_m/dx) in Eq. (8) is represented in the GSE generated system sensitivity derivatives $(d\boldsymbol{y}/d\boldsymbol{x})$ when constraints g_m are modeled as state variables y.

With equality constraints h modeled as state variables y, they can be approximated using Eq. (7) (nonlocal) or Eq. (6) (local). The respective approximations of the full vector of equality constraints $\boldsymbol{h}$ can be imposed in each subspace optimization.

The approximation of state variables based on the GSE system sensitivity derivatives allows the subspace optimizations to be temporarily decoupled and optimized concurrently (Fig. 2). Imposing the constraint approximations of the nonlocal subspaces [Eqs. (9) and (7)] is an attempt to maintain system coupling during local subspace optimization [Eq. (4)]. Therefore, design vectors visited during subspace optimizations should tend toward maintaining system coupling. The global approximation procedure used in this research uses the information obtained during the subspace optimizations and accumulated in the design data base. The design data base stores the objective function and cumulative constraint values associated with design vectors investigated during the subspace optimizations. In Ref. 2 and in this research, the design site results from each line search in the generalized reduced gradient (GRG) subspace optimizer are added to the design data base. A system analysis is applied at each subspace solution vector providing exact information to the design data base. The subspace solution vectors are then more heavily weighted as part of a least squares solution[18] in forming the second-order global approximation of the coordination procedure of global approximation.

After the concurrent subspace optimizations, a second-order approximation to the global problem (coordination procedure, Fig. 2) is formed about $(\boldsymbol{x}^0, \boldsymbol{y}^0)$ as described in the next section. The solution of this approximation problem $\boldsymbol{x}^*$ is used as the next design iterate. At the new design vector $\boldsymbol{x}^*$, a solution to the coupled equations (3) is obtained as the new state vector $\boldsymbol{y}^*$. The objective function and system constraints are evaluated at $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ and added to the design data base. Based on this design $(\boldsymbol{x}^*, \boldsymbol{y}^*)$, the GSE system sensitivity derivatives are calculated (system coupling updates, Fig. 2) for use in the next round of concurrent subspace optimizations and in the coordination procedure of global approximation. The respective processes of system-coupling updates, concurrent subspace optimizations, and the coordination procedure of global approximation are repeated until no significant change in the design vector $\boldsymbol{x}$ is observed.

The descent properties of the subspace optimizations serve indirectly to guide overall system convergence, while providing data for the design data base of global approximation. Optimization of the global approximation problem serves as the primary driver to direct overall system convergence to an optimum. The global format of approximation provides a design tradeoff feature for the algorithm.

The global approximation procedure can return an infeasible design $(\boldsymbol{x}^*, \boldsymbol{y}^*)$. This creates a problem if a subspace optimizer cannot locate a feasible starting vector within the confines of the move limit restrictions of Eq. (4). In these cases the violated constraints are reformulated using constraint improvement goals. These improvement goals provide a similar feature to that imposed by the responsibility factor in Ref. 9. Basically the improvement goal allows for temporary constraint relaxation at the subspace optimizations. The relaxation of violated constraints allows each subspace optimizer to locate a feasible starting point within the local move limit restrictions. Equation (10) depicts a violated cumulative constraint modeled for 50% improvement.

$$(KS) - (1.0 - 0.50)KS^0 \geq 0 \tag{10}$$

where KS^0 is in violation.

When using Eq. (10), the constraint values generated during the subspace optimizations must be corrected before transfer to the design data base so as to reflect their approximate value (i.e., KS). Constraint improvement goals allow the subspace optimizations to transfer information obtained in the infeasible design space to the design data base. The coordination procedure of global approximation uses this information to avoid infeasible design regions. The improvement goals differ from the formulation in Ref. 9 in that they are not used to drive constraint satisfaction or the overall system optimization. The improvement goals allow for the building of a more complete design data base during the subspace optimizations. It is the coordination procedure of global approximation that drives constraint satisfaction and overall system optimization.

IV. Modified Coordination Procedure of Global Approximation

After the concurrent subspace optimizations, a global approximation problem is formulated about the current design vector using information stored in the design data base. The global approximation problem models the objective function and cumulative constraints using approximating functions based on real design data obtained during the subspace optimizations (i.e., design data base). Use of approximating functions eliminates the computationally intensive calculations of function and constraints normally required in an optimization of the full problem.

The idea is to exploit the design data generated in the concurrent subspace optimizations. This mirrors the approach used in actual design environments, where individual design groups operate concurrently followed by a series of design tradeoffs and compromises. These tradeoffs and compromises are made based on the individual design groups' experience and information base. In the proposed algorithm, the design data base mirrors the designers' experience and known data, whereas optimization of the global approximation problem facilitates design tradeoffs.

This coordination procedure of global approximation is significantly different than the sensitivity based coordination procedure (COP) in Ref. 9. In the COP approach, constraint responsibility and tradeoff factors are used to coordinate subspace optimizations. The updated design vector of Ref. 9 is the simple combination of local optimal design subvectors (i.e., simultaneous solution). The simultaneous solution does not necessarily represent an improved or feasible solution as shown in Ref. 6. The modified coordination procedure of global approximation proposed in this research is designed to more fully exploit the information obtained during subspace optimizations. The optimal solution of the global approximation problem is proposed as a more robust update of the design vector for the subsequent round of subspace optimizations.

The approximating functions used in Ref. 2 were based on the Rasmussen[14] accumulated approximation strategy. The adaptation of Renaud and Gabriele[2] of accumulated approximation incorporates a first-order basis (GSE) with interpolation features activated at or near previously visited design sites.

In this study the accumulated approximation is modified to include a second-order basis of approximation. This second-order information can be obtained numerically using data currently available in the design data base. A quadratic polynomial approximation to the design is formed using the strategy of Vanderplaats.[18] This polynomial is used as the basis function for accumulated approximation replacing the linear basis used in Ref. 2. The interpolating features of the approach of Rasmussen[14] remain the same except that gradient information is no longer incorporated in the interpolation terms.

Equation (11) depicts the kth accumulated approximation $P_k(\boldsymbol{x})$ of a generic function $F(\boldsymbol{x})$ based on $(k-1)$ previously visited design vectors. The basis function of accumulated approximation is $L_k(\boldsymbol{x})$, and $P_k(\boldsymbol{x})$ approaches the value of $L_k(\boldsymbol{x})$ away from previously visited design vectors and interpolates to known function values $F(\boldsymbol{x}_p)$ at or near previously visited design sites $(\boldsymbol{x}_p)$. The influence functions ϕ_p smooth between the basis function $L_k(\boldsymbol{x})$ and the interpolation(s) $F(\boldsymbol{x}_p)$.

$$P_k(\boldsymbol{x}) = \frac{L_k(\boldsymbol{x}) \prod_{p=1}^{k-1} [1 - \phi_p(\boldsymbol{x})] + [1 - \phi_k(\boldsymbol{x})] \sum_{p=1}^{k-1} [\phi_p(\boldsymbol{x}) F(\boldsymbol{x}_p)]}{\prod_{p=1}^{k-1} [1 - \phi_p(\boldsymbol{x})] + [1 - \phi_k(\boldsymbol{x})] \sum_{p=1}^{k-1} \phi_p(\boldsymbol{x})} \tag{11}$$

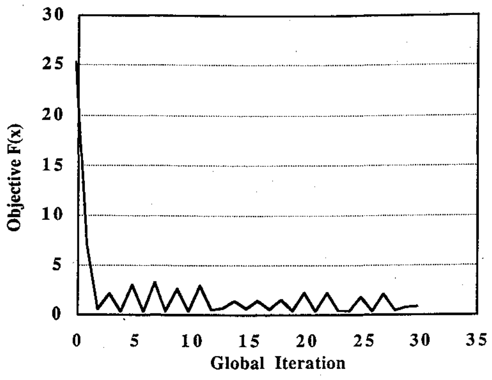

Fig. 4 First-order convergence history, Rosenbrock's function.

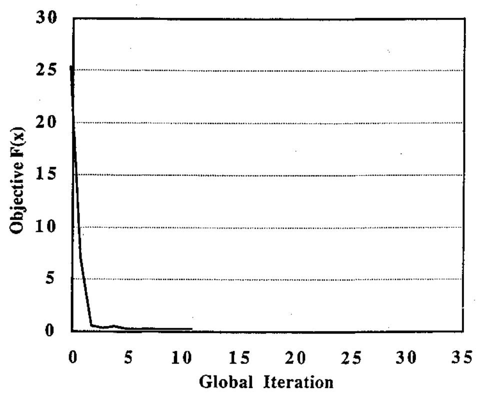

Fig. 5 Second-order convergence history, Rosenbrock's function.

The basis function $L_k(x)$ in the Ref. 2 study was a first-order Taylor series approximation based on GSE sensitivities. This first-order basis function is

$$L_k(x) = F(x_k) + \nabla F(x_k)\Delta x \quad 12$$

where $\nabla F(x_k)$ are the global sensitivities determined from the GSE. In this study, $L_k(x)$ is formulated as a quadratic polynomial approximation to the design. Equation (13) depicts the quadratic polynomial approximation used in this study:

$$L_k(x) = F(x_k) + \nabla F(x_k)\Delta x + \tfrac{1}{2}\Delta x^T H(x_k)\Delta x \quad (13)$$

The design data base stores randomly spaced design information generated by the subspace optimizations. Using this data and the global sensitivities for $\nabla F(x_k)$ one can solve for the second-order terms $H(x_k)$ using the strategy of Vanderplaats.[18] Since the design data is randomly spaced, the approximation is not a second-order Taylor series but is a quadratic polynomial approximation to the design. For a problem of n dimensions, a total of $q = n(n+1)/2$ design sites need to be visited to solve for all of the second-order terms. In implementation studies, the subspace optimizations are seen to generate sufficient design site information. When the size of the design data base exceeds q design sites, a weighted least squares fit is used. The current subspace solution design vectors are weighted to hold more influence in the second-order approximation of $H(x_k)$.

The reader is referred to Ref. 2 for additional details about the accumulated approximation $P_k(x)$ and the influence function ϕ. Note that the gradient augmentation of Ref. 2 is removed in the interpolation terms. With a second-order basis, the gradient augmentation of interpolation terms is deemed unnecessary.

V. Results

Strict implementation of the nonhierarchic optimization procedure of Ref. 2 applied to a nonhierarchic decomposition of Rosenbrock's function failed to converge to the global optimum. Rosenbrock's function,[15] a two variable problem subject to two inequality constraints, is given in Eq. (14). The problem is decomposed into two design subspaces, $\{x_1, KS_1\}$ and $\{x_2, KS_2\}$, respectively:

$$F(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \quad (14)$$

subject to

$$KS_1(g_1) \geq 0.0 \text{ and } KS_2(g_2) \geq 0.0$$

where

$$g_1 = 2x_1 - x_2^2 - 1.0 \geq 0.0$$

and

$$g_2 = 9.0 - 0.8x_1^2 - 2x_2 \geq 0.0$$

The first-order nature of the Rasmussen[14] accumulated approximations used in Ref. 2 led to a cycling condition in the first-order coordination procedure of global approximation. Rosenbrock's function is not representative of the size of the problem for which the proposed algorithm is intended. The algorithm is proposed for large complex design problems where decomposition is required in the solution process. The nonhierarchic decomposition of Rosenbrock's function does, however, provide a visual check of algorithm performance in initial studies. By modifying the accumulated approximation strategy to include a second-order basis, the cycling problem is eliminated. Figures 4 and 5 plot the convergence characteristics of the algorithm of Renand and Gabriele[2] on the Rosenbrock's function for the first- and second-order methods, respectively. The improved performance at convergence can be traced to the increased accuracy of the second-order-based approximation. The unique nonlinear features of Rosenbrock's function were not handled robustly by the first-order-based approximations of the proposal in Ref. 2. The inequality constraints remain inactive in this study.

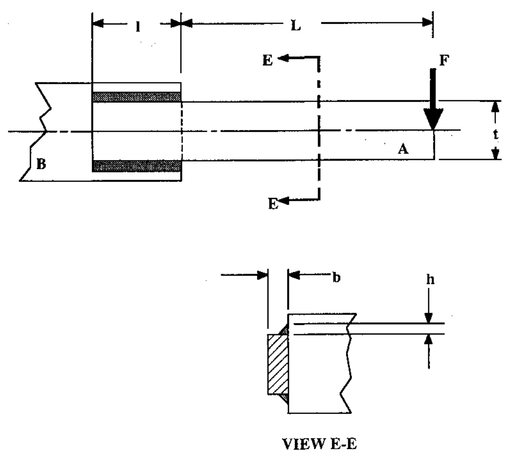

Fig. 6 Welded beam problem.

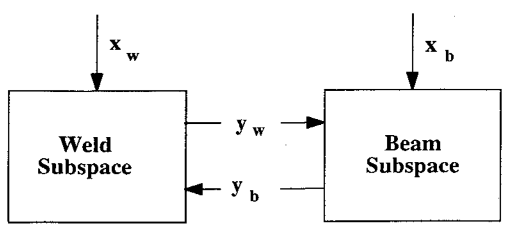

Fig. 7 Welded beam: nonhierarchic network structure.

Applied to a more robust test problem,[16] the second-order-based approximations are observed to improve the accuracy of cumulative constraint approximations. The proposal in Ref. 2 uses the Kreisselmeier/Steinhauser[17] cumulative constraint function [i.e., KS function, Eq. (5)]. A cumulative constraint is by nature difficult to approximate, in part due to active set changes occurring in the constraint family. In the investigation in Ref. 2, the welded beam problem of Ragsdell and Phillips[16] and the speed reducer problem of Golinski[19] were successfully handled by the first-order-based approximations of the proposed coordination procedure. It was noted in that study that although the Rasmussen[14] accumulated first-order-based approximation was very accurate in approximating the objective function, it was less robust at approximating the cumulative constraints. In this study, the modified second-order basis of the accumulated approximation of Rasmussen[14] provides a more accurate approximation of cumulative constraints as implemented in a nonhierarchic decomposition of the welded beam problem. The more accurate cumulative constraint approximations produce improved convergence.

Figure 6 details the familiar welded beam of Ragsdell and Phillips,[16] and Fig. 7 depicts the nonhierarchic network structure imposed in Ref. 2 and repeated in this study. The design variables in the problem are the dimensions h, l, t, and b as shown in Fig. 6. The nonhierarchic decomposition imposed in this problem defines a weld subspace (h, l, KS_1) and a beam subspace (t, b, KS_2). The objective function for manufacturing cost incorporates all four design variables. The five constraints include weld and beam stress limits along with weld size, beam buckling, and deflection limits. Complete nonhierarchic decomposition details for the welded beam problem are included in Ref. 2.

Table 1 compares the optimization results of three optimization trials for the welded beam problem. The first- and second-order

Table 1 Comparative results welded beam optimization

	Nonhierarchic optimization (second order)	Nonhierarchic optimization (first order)	GRG, $\rho = 100$
Objective $F(x)$	2.40	2.43	2.44
h	0.239	0.2475	0.2268
l	6.321	6.2743	6.895
t	8.371	8.285	8.296
b	0.243	0.2475	0.2451
KS_1	−0.000884	−0.00116	−0.0000189
$(g_1) - y_1$	0.000800	0.018771	−0.0000132
$(g_3) - y_2$	0.017659	0.000305	0.07465531
KS_2	−0.000023	0.0108	−0.0004300
$(g_2) - y_3$	0.018382	0.011468	0.0042454
$(g_4) - y_4$	0.001387	0.037916	0.0094193
$(g_5) - y_5$	0.938752	0.937639	0.9372699
System analysis required (constraints)	114	150	301

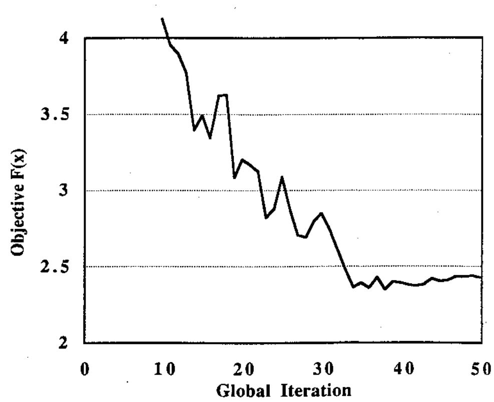

Fig. 8 Objective convergence history for first-order case: welded beam.

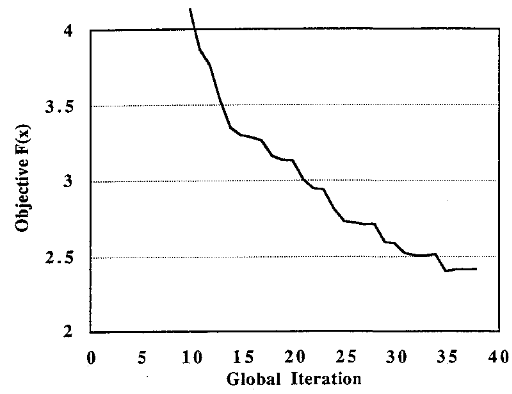

Fig. 9 Objective convergence history for second-order case: welded beam.

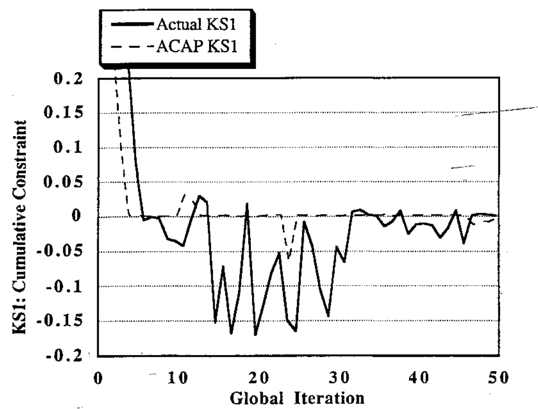

Fig. 10 Cumulative constraint for first-order case: welded beam.

implementations of the algorithm of Renaud and Gabriele[1,2] are compared with results obtained using a conventional GRG optimizer (OPT3.2) applied to the full system. The cumulative constraints KS have been normalized to the range $-1.0 \leq KS \leq 1.0$, where a value greater than or equal to zero is feasible. A cumulative constraint parameter of $\rho = 100$ is used along with a Rasmussen parameter $\alpha = 0.01$ in this study. Move limits of 10% are imposed at the subspace optimizations and in the coordination procedure of global approximation.

Because of less cycling at convergence, 24% fewer system analyses are required as compared with the first-order coordination procedure of Renaud and Gabriele.[2] The second-order strategy requires 62% fewer system analyses as compared with the conventional GRG optimizer. The goal in multidisciplinary design optimization is to reduce the number of system analyses required for system optimization.

Figures 8 and 9 plot the convergence characteristics of the welded beam objective function for the case of first- and second-order-based objective function and cumulative constraint approximations. In the first-order case, the objective cycles during optimization due to inaccuracies in the cumulative constraint predictions. Use of the second-order-based approximations leads to the monotonic convergence plot in Fig. 9 and the reduction in system analyses required.

Figures 10 and 11 compare the accuracy of cumulative constraint approximation for both the first- and second-order-based studies. The plots in Figs. 10 and 11 compare the final accumulated approximation of cumulative constraint (ACAP KS) value with the actual cumulative constraint value (actual KS) for one of the cumulative constraints in the welded beam study. The final approximation of cumulative constraint (ACAP KS) is taken from the coordination procedure's approximate global optimization

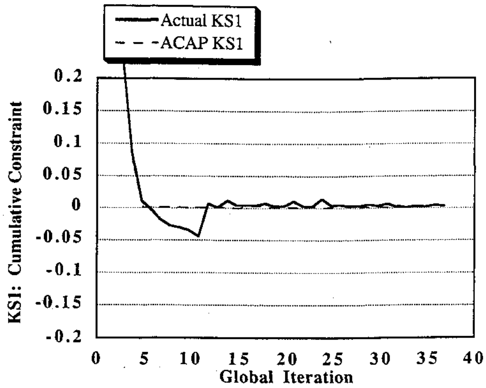

Fig. 11 Cumulative constraint for second-order case: welded beam.

solution. This approximate value is compared with the actual cumulative constraint value (actual *KS*) which is calculated as part of the system coupling updates.

VI. Conclusions

In initial tests the modified coordination procedure of global approximation results in improved convergence as compared with the authors' first-order implementation of the nonhierarchic system optimization strategy. In this study the coordination procedure of Renaud and Gabriele[2] is modified to include a second-order basis using existing data available from subspace optimizations. The modification provides improved accuracy in the approximation of objective function and cumulative constraints. Improved accuracy of approximation reduces cycling in the system optimization procedure. Because of less cycling at convergence, a significant reduction in the number of system analyses required for optimization is observed.

The proposed algorithm accommodates existing design practices where design teams operate concurrently, restricted by periodic design tradeoffs based on prior design experience. In this study, the subspace optimizations provide for individual design teams working concurrently. Information obtained during the nonlinear subspace optimizations builds a design data base representative of design experience. The modified coordination procedure of global approximation, built from the design data base, accommodates design tradeoffs and provides robust algorithm coordination in initial studies.

It should be noted that in traditional optimization literature second-order methods are generally considered computationally expensive. In the proposed algorithm, the decision to accommodate existing design practice through use of concurrent subspace optimizations inherently produces the required second-order computations. The proposed algorithm simply attempts to exploit the information obtained during subspace optimizations. The use of available data to form second-order approximations that improve optimization is supported in a recent study by Bennett and Park.[20]

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